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PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 61.

February 8th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Rev. Henry Tibbs and John O'Donovan, Esq., were elected members of the Academy.

A paper by the Rev. M. Roberts, on the lines of curvature on the surface of the Ellipsoid, was read by Mr. Ingram.

The analogy between the lines of curvature on an ellipsoid, and a system of homofocal plane or spherical conics, was first remarked by Mr. M. Roberts, in a note communicated by M. Liouville to the French Academy of Sciences, where he has shewn, among other things, that the sum (or difference) of the geodetic distances from the umbilics to any point of the same line of curvature is constant. The following properties, which he has recently obtained by assigning a significant geometrical meaning to the constant introduced by M. Jacobi in the integral of the differential equation of the geodetic line, appear worthy of the attention of geometers:

1. Assuming the distance between the umbilics (interior) as the base of a system of triangles, whose sides are geodetic

lines, and of which the product of the tangents of the semi-angles at the base is constant, the locus of the vertex will be a line of curvature.

2. And if the ratio of the tangents of the aforesaid angles be constant, the locus of the vertex will be a line of curvature of the orthogonal system.

3. As the arc s of a plane curve is expressed in polar coordinates by the equation

$$ds^2 = d\rho^2 + \rho^2 d\omega^2,$$

and the arc of a spherical curve by the equation

$$ds^2 = d\rho^2 + \sin^2\rho\, d\omega^2,$$

so let the arc of a curve on the surface of an ellipsoid, referred to the geodetic distance (ρ) from one of the umbilics, and the angle (ω) made by ρ with the section containing the umbilics, be given by the equation

$$ds^2 = d\rho^2 + \mathrm{P}^2 d\omega^2,$$

and, as Mr. Roberts has demonstrated,

$$\mathrm{P}\sin\omega$$

will be the perpendicular distance of the point (ρ, ω) from the plane of the umbilics. Hence, P', ω' denoting the same things for the contiguous umbilic, we have

$$\mathrm{P}\sin\omega = \mathrm{P}'\sin\omega', \text{ or } \frac{\mathrm{P}}{\mathrm{P}'} = \frac{\sin\omega'}{\sin\omega},$$

which may be regarded as an extension, to the surface of an ellipsoid, of the fundamental property of plane and spherical triangles, that the sides (or the sines of the sides) are proportional to the sines of the opposite angles.

4. Let ω be a right angle, and the corresponding geodetic vector will pass through the vertex of the mean axis, and its length comprised between this point and the origin (the umbilic) will be equal to the quadrant of the elliptic section containing the umbilics. The function P of this arc will be equal to the mean semiaxis of the surface, in the same way as the sine of the quadrant is the radius.

5. Let geodetic lines issuing from the same point upon a line of curvature, and passing through the umbilics O, O′, meet the line of curvature again in the points P, P′. Then will the locus of the point of concourse of the geodetic lines OP′, O′P, be a line of curvature of the same species as the given one.

The following note, by Mr. M. Roberts, on a theorem relating to the Hyperbola, was also read:

Let S denote the difference between the infinite arc and the asymptote of the hyperbola, whose equation is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\,;$$

and let S′ be the length of the quadrant of the curve which is the locus of the feet of perpendiculars dropped from the centre upon its tangents; also, let Σ, Σ′ denote the same things in reference to the conjugate hyperbola

$$\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1,$$

and we shall have

$$SS' + \Sigma\Sigma' = \tfrac{1}{2}\pi\left\{\frac{a^3}{b} - \sqrt{a^2 - b^2}\,s\right\}$$

where we suppose $a > b$, and denote by s an arc of the first hyperbola, measured from the vertex to the point whose coordinates (x', y') are

$$x' = \frac{a^2}{b}, \qquad y' = \sqrt{a^2 - b^2}.$$

If $a = b$ the hyperbola is equilateral; the derived curve is the common lemniscate, $S = \Sigma$, $S' = \Sigma'$; and

$$SS' = \tfrac{1}{4}\pi a^2,$$

a theorem proposed by Mr. W. H. Talbot, and proved by M. Sturm, in vol. xiv. of Gergonne's *Annales de Mathematiques*, page 17.

Professor Harrison read the following paper on the anatomy of the elephant:

"Having had, within the last few weeks, an opportunity of examining the body of an elephant which died in this city, volume appear to be part of a translation into Irish, from some other language, of the ancient history of the Greeks, Romans, Persians, &c. These leaves are not in the same handwriting as the preceding treatise on medicine, nor do they appear to have been at any time part of the volume. The second page is marked Chapter III., between which and Chapter VI., on the fifth page following, no other chapter intervenes, so the leaves are not consecutive. The style of the translation is that of the early part of the sixteenth century."

The fragment now presented begins with the end of Chapter I. It contains the whole of Chapter II., and the beginning of Chapter III.

The following is a translation of the heading of Chapter II.:

"The second chapter, in which Belus, Ninus, and Semiramis, are spoken of, and of their successors in the kingdom of Babilon, until the fifth year of the reign of Sethos."

The chapter then opens in words to this effect: "Syncellus says, following the opinions of Halanicus, Ctesias, Halicarnassus, and Hephalion, that Abraham was fourteen years dwelling in the country of Canaan, when Belus came to conquer Babilon."

Page 7 contains a list of the Judges and Chiefs of the Children of Israel, and of the contemporary kings of Egypt.

The third chapter: "Of the kings of Asiria from the time of Laostines to the nineteenth year of the reign of Nabuchadonosor, who plundered and destroyed the Temple."

Presented by Eugene Curry, Esq.

ERRATA IN PRECEDING NUMBER.

Page 383, line 3, *for* "the Rev. M. Roberts" *read* "Mr. M. Roberts."
„ 385, „ 6, *for* "Mr. M. Roberts" *read* "the Rev. W. Roberts."
„ 385, „ 15, *for* $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$, *read* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.